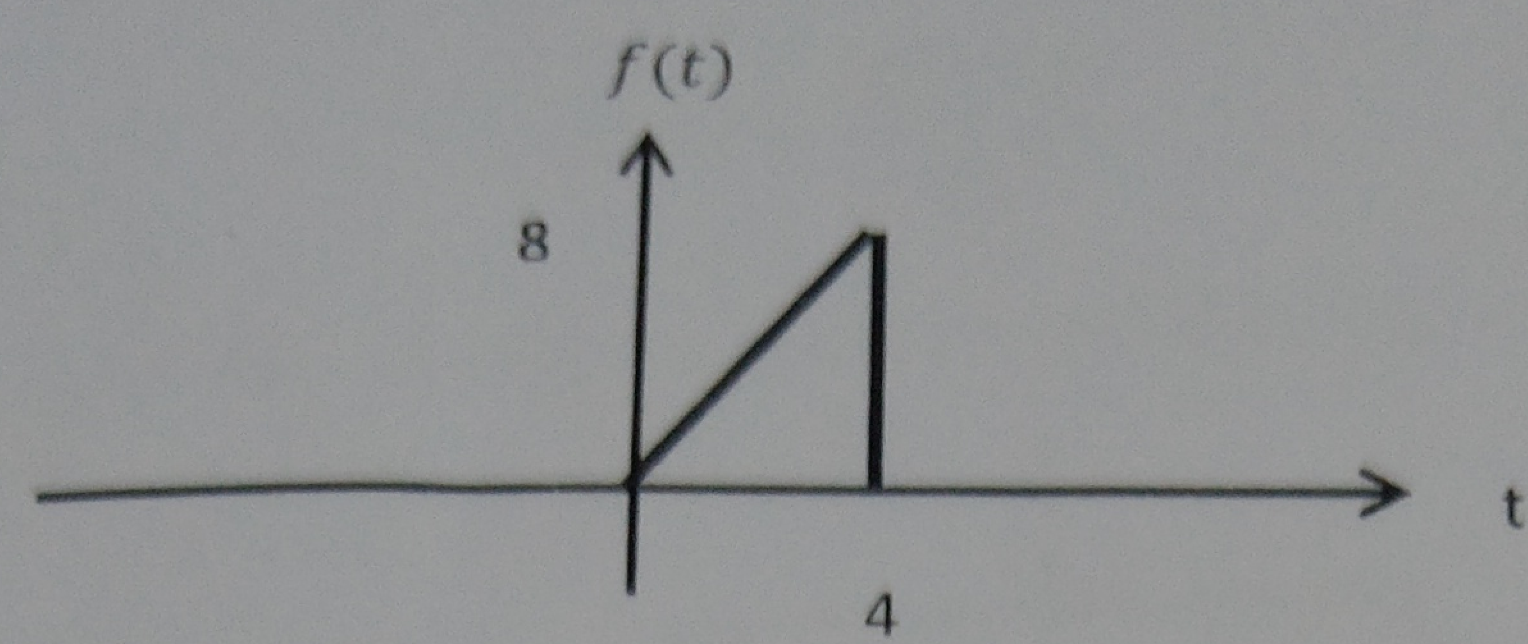
Fourier Transform:

س-15- باستخدام خاصية التفاضل اوجد تحويلة فوريير للإشارة بالشكل



س-16 – باستخدام خاصية الـــ Duality اوجد تحويلة فوريير للإشارة

$$x(t) = \frac{\sin 2t}{\pi t}$$

س-17- باستخدام جداول التحويل وجداول الخواص ارسم الطيف الترددي للإشارات التالية:

a. $s(t) = sin(2t + \frac{\pi}{2})\cos(8t)$

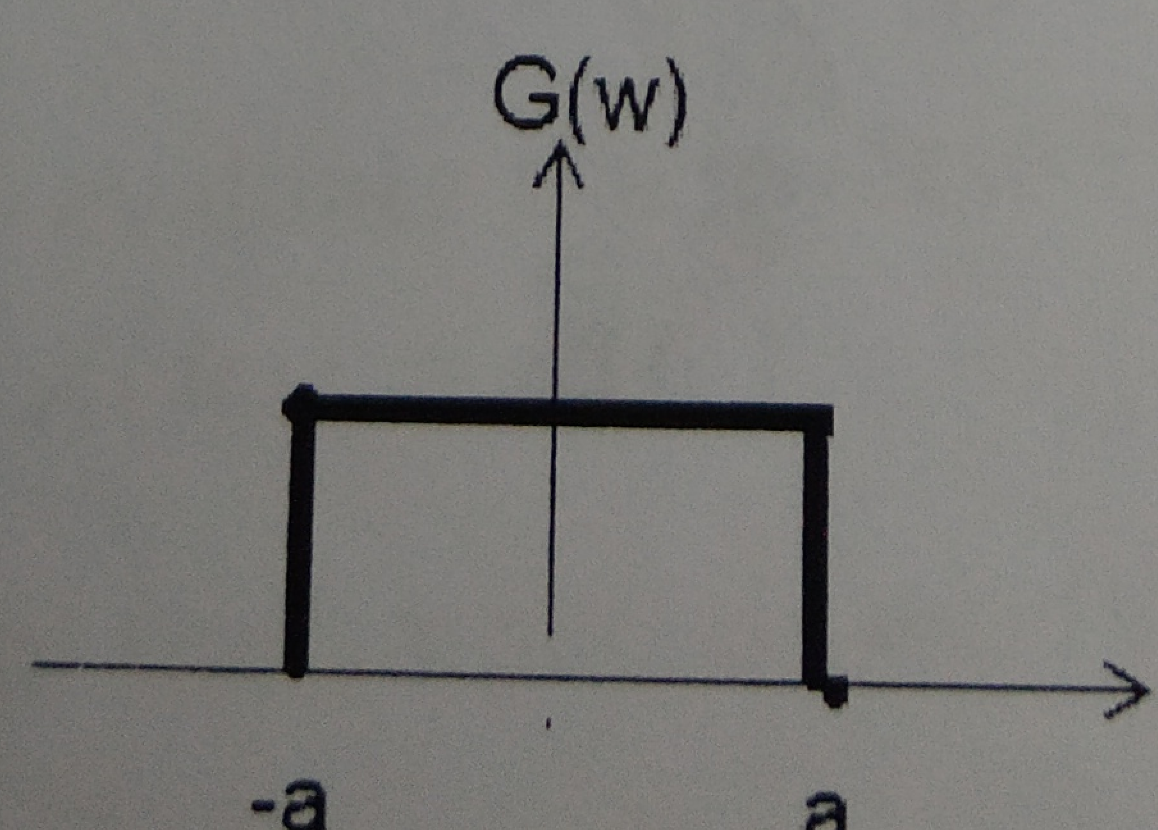
b. $m(t) = 3u(t+4) - 3u(t-4)$

c. $f(t) = sgn(t) + u(-t)$

Q(18) For the shown signal:

a. Write an expression for $G(j\omega)$

b. Find $g(t)$

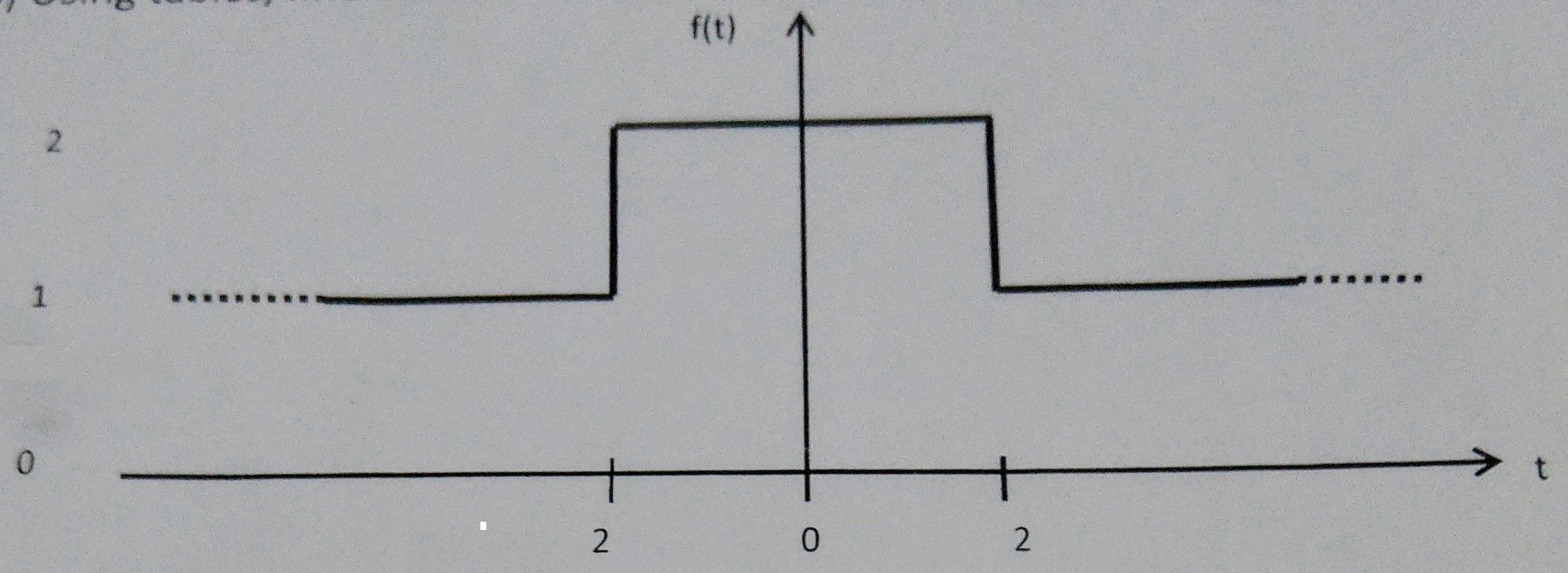


س-19- اذا كان $G(j\omega) = \frac{4}{j\omega+6}$

اوجد تحويلة فورير للاشارة:

$$x(t) = \frac{d^2 g(t)}{dt^2}$$

Q(20) Using tables, find the Fourier Transform of the shown non periodic signal?



Q(21) - Using tables, find the Fourier Transform of $y(t) = f(t).\cos(100t)$ where

$$f(t) = \frac{2}{4+jt}$$

س-22- باستخدام جداول التحويل وجداول الخواص. اوجد تحويلة فورير لكل من:

a. $f(t) = u(t+4) - u(t+2) + u(t-2) - u(t-4)$

b. $x(t) = \frac{\sin 2t}{\pi t}$

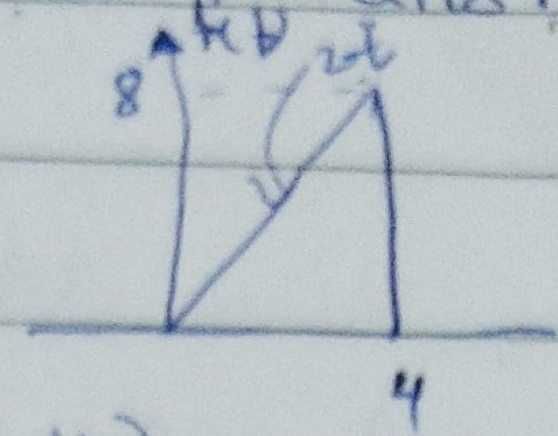
Q(22) Use duality to evaluate the inverse FT of the unit step function in frequency domain $X(j\omega) = u(\omega)$

Q(23) Find the FT of the amplitude modulated signal which given by

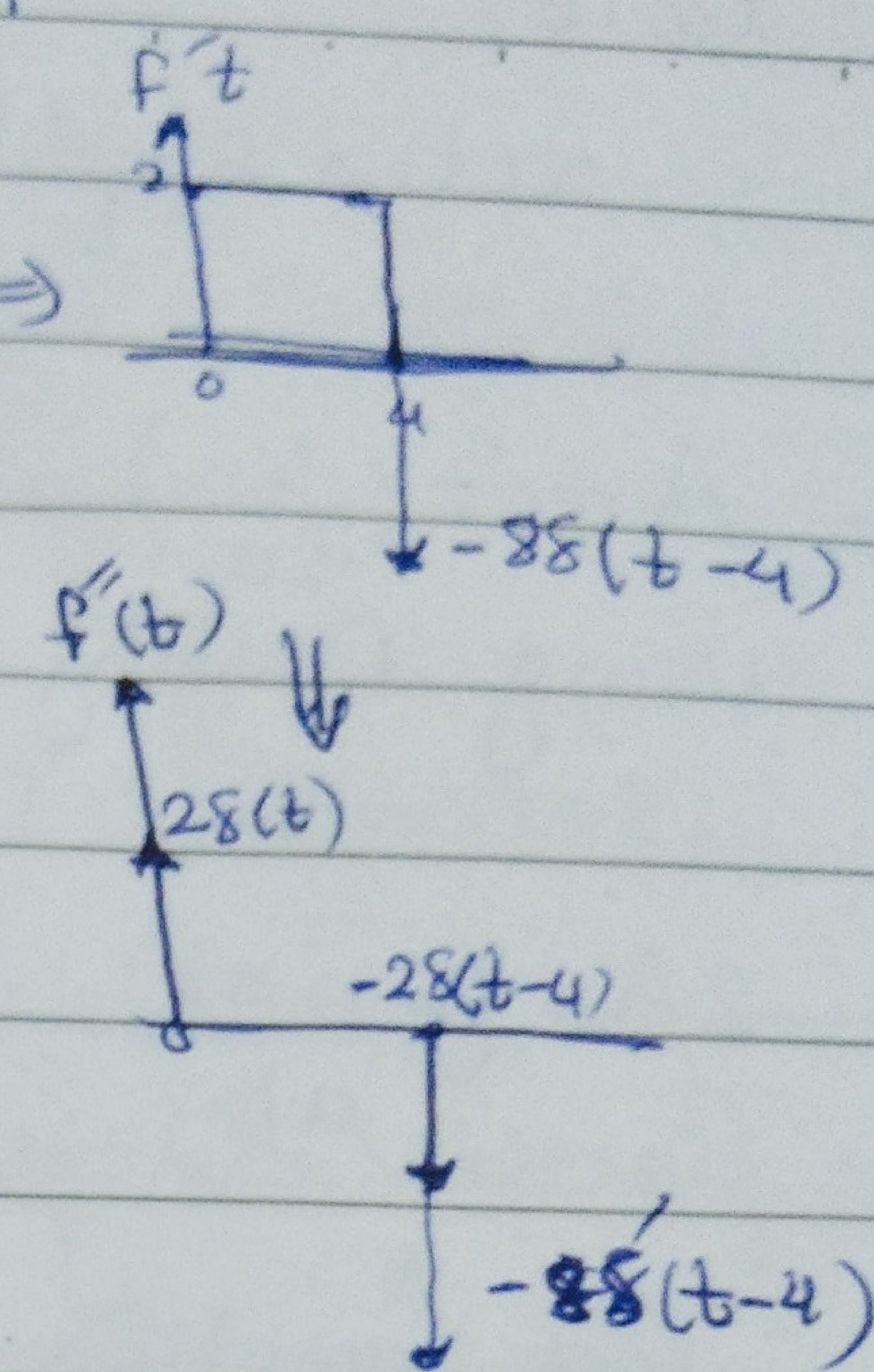
$y(t) = A[1 + km(t)]\cos(\omega_o t)$ in terms of the CTFT $M(\omega)$ of the information signal $m(t)$

Fourier Transform

Q15 :-



$\Rightarrow$



$f(t) = 2r(t) - 2r(t-4) + 8u(t-4)$

$f'(t) = 2u(t) - 2u(t-4) - 8\delta(t-4)$

$f''(t) = 2\delta(t) - 2\delta(t-4) - 8\delta'(t-4)$

Taking Fourier transform for both sides

$(jw)^2 F(w) = 2 - 2e^{-4jw} - 8jwe^{-4jw}$

$$F(w) = \frac{2 - 2e^{-4jw} - 8jwe^{-4jw}}{j^2 w^2}$$

$$F(w) = \frac{2e^{-4jw} - 2 + 8jwe^{-4jw}}{w^2} \quad \#$$

Another solution :-

Or $\Rightarrow F(w) = \int_0^4 2t\, e^{-jwt}\, dt$

$u = t \quad dv = e^{-jwt}\, dt$

$du = dt \quad v = \frac{e^{-jwt}}{-jw}$

$$F(w) = 2\left[\frac{-te^{-jwt}}{jw}\Bigg|_0^4 + \int_0^4 \frac{e^{-jwt}}{jw}\right]$$

$$F(w) = 2\left[\frac{-te^{-jwt}}{jw} - \frac{e^{-jwt}}{j^2w^2}\right]\Bigg|_0^4$$

$$F(w) = \frac{-8e^{-4jw}}{jw} - \frac{2e^{-4jw}}{j^2w^2} + \frac{2}{j^2w^2}$$

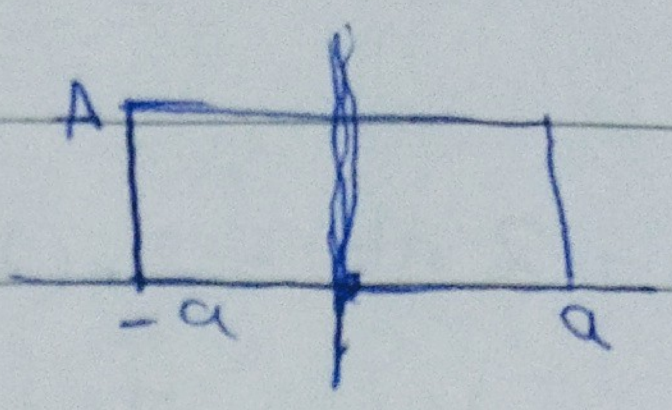
$$= \frac{-8jwe^{-4jw} - 2e^{-4jw} + 2}{j^2w^2}$$

$$= \frac{2e^{-4jw} - 2 + 8jwe^{-4jw}}{w^2} \quad \#$$

$=$

Q16: -

$x(t) = \frac{\sin 2t}{\pi t}$



$A\,\text{rect}\left(\frac{t}{2a}\right) \xrightarrow{F.T} 2Aa\,\frac{\sin(\omega a)}{\omega a}$ -

Using Duality property:-

$2Aa\left(\frac{\sin at}{at}\right) \xrightarrow{F.T} 2\pi \times A\,\text{rect}\left(\frac{-\omega}{2a}\right)$

rect function is an even signal so $\text{rect}(-\omega) = \text{rect}(\omega)$

$\cancel{2}\cancel{A}\cancel{a}\,\frac{\sin(at)}{\cancel{a}t} \xrightarrow{F.T} \cancel{2}\pi \times \cancel{A}\text{rect}\left(\frac{\omega}{2a}\right)$

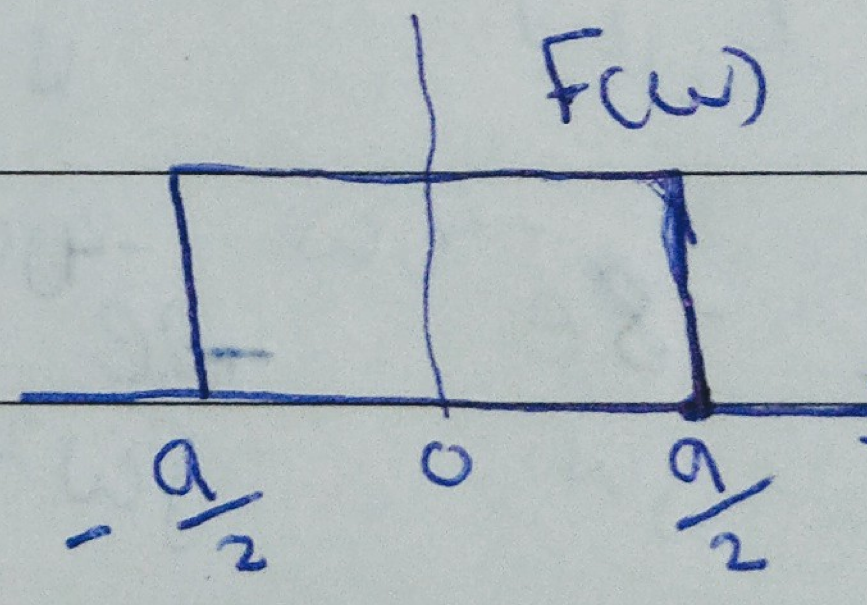
$\frac{\sin(at)}{t} \xrightarrow{F.T} \pi\,\text{rect}\left(\frac{\omega}{2a}\right)$

when $a=2$

$\frac{\sin(2t)}{t} \xrightarrow{F.T} \pi\,\text{rect}\left(\frac{\omega}{a}\right)$

$\frac{\sin(2t)}{\pi t} \xrightarrow{F.T} \text{rect}\left(\frac{\omega}{a}\right)$ ##

* To varify we can use the inverse Fourier transform



$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\,e^{j\omega t}\,d\omega$

$f(t) = \frac{1}{2\pi}\int_{-2}^{2} e^{j\omega t}\,d\omega = \frac{1}{2\pi j t}\,e^{j\omega t}\Big|_{-2}^{2} = \frac{1}{\pi t\,2j}\left[e^{2jt} - e^{-2jt}\right]$

$f(t) = \frac{1}{\pi t}\left[\frac{e^{2jt} - e^{-2jt}}{2j}\right] = \frac{\sin(2t)}{\pi t}$

Q17 :-

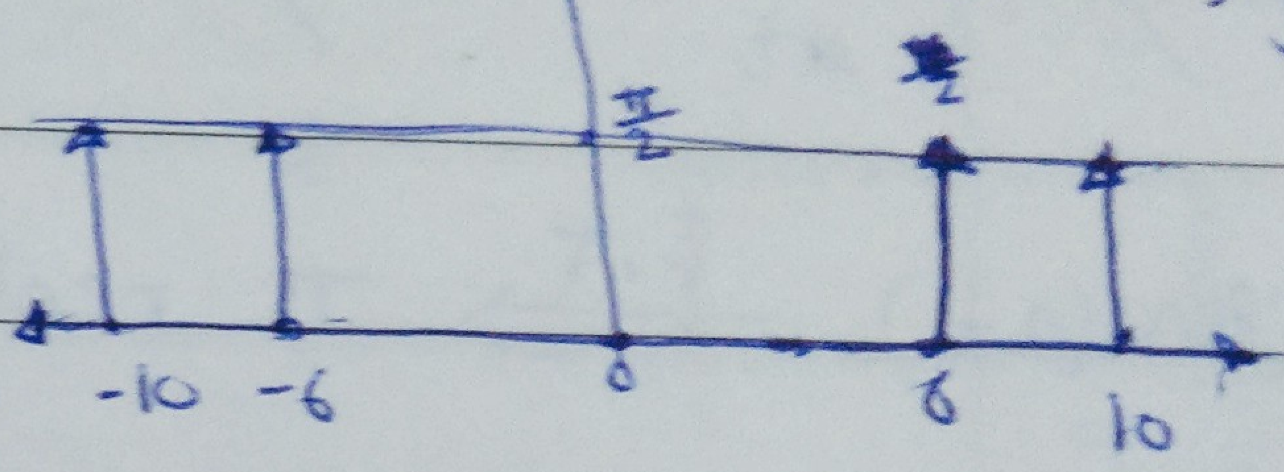
(a) $s(t) = \sin\left(2t + \frac{\pi}{2}\right) \cos(8t)$, $\sin(\theta) = \cos\left(\theta - \frac{\pi}{2}\right)$

$s(t) = \cos(2t)\cos(8t)$

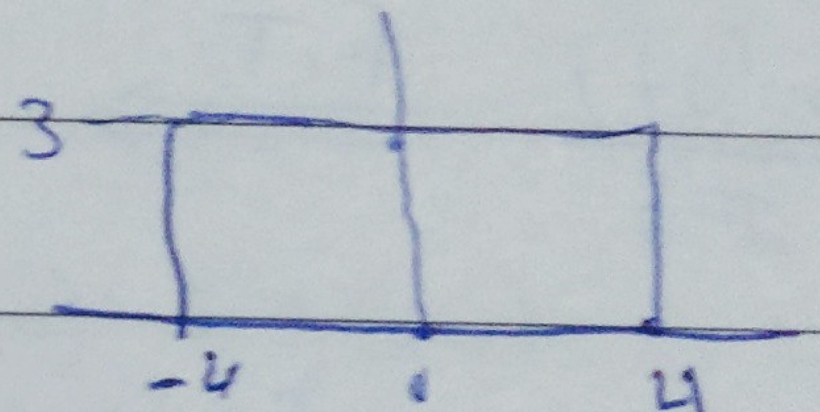
$\cos(2t) \xrightarrow{F.T} \pi\left[\delta(\omega-2) + \delta(\omega+2)\right]$

$\cos(2t)\cos(8t) \xrightarrow{F.T} \frac{\pi}{2}\left[\delta(\omega-2-8) + \delta(\omega+2-8) + \delta(\omega-2+8) + \delta(\omega+2+8)\right]$

$\cos(2t)\cos(8t) \xrightarrow{F.T} \frac{\pi}{2}\left[\delta(\omega+10) + \delta(\omega+6) + \delta(\omega-6) + \delta(\omega-10)\right]$ #



(b) $m(t) = 3u(t+4) - 3u(t-4)$

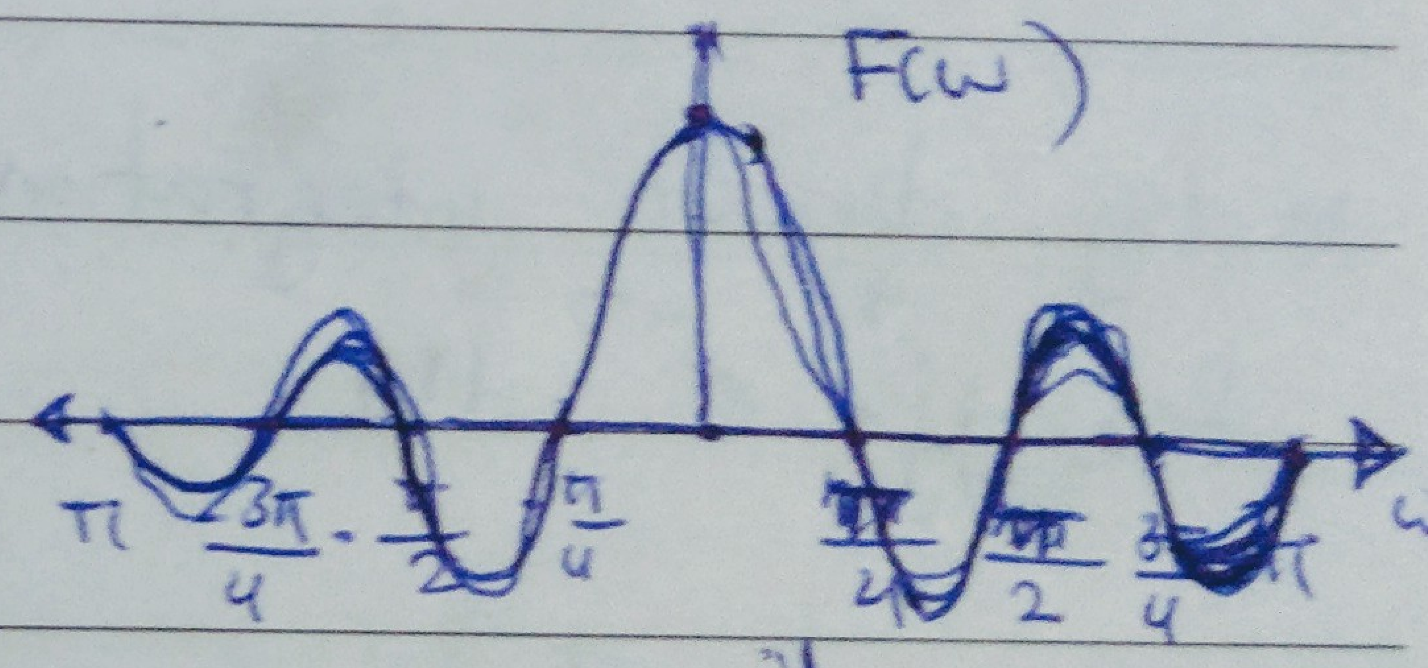


$m(t) = 3\,\mathrm{rect}\left(\frac{t}{8}\right)$, $A = 3$, $a = 4$, $2a = 8$

$A\,\mathrm{rect}\left(\frac{t}{2a}\right) \xrightarrow{F.T} 2Aa \frac{\sin(\omega a)}{\omega a}$

$3\,\mathrm{rect}\left(\frac{t}{8}\right) \xrightarrow{F.T} 2 \times 3 \times 4 \frac{\sin(4\omega)}{4\omega}$

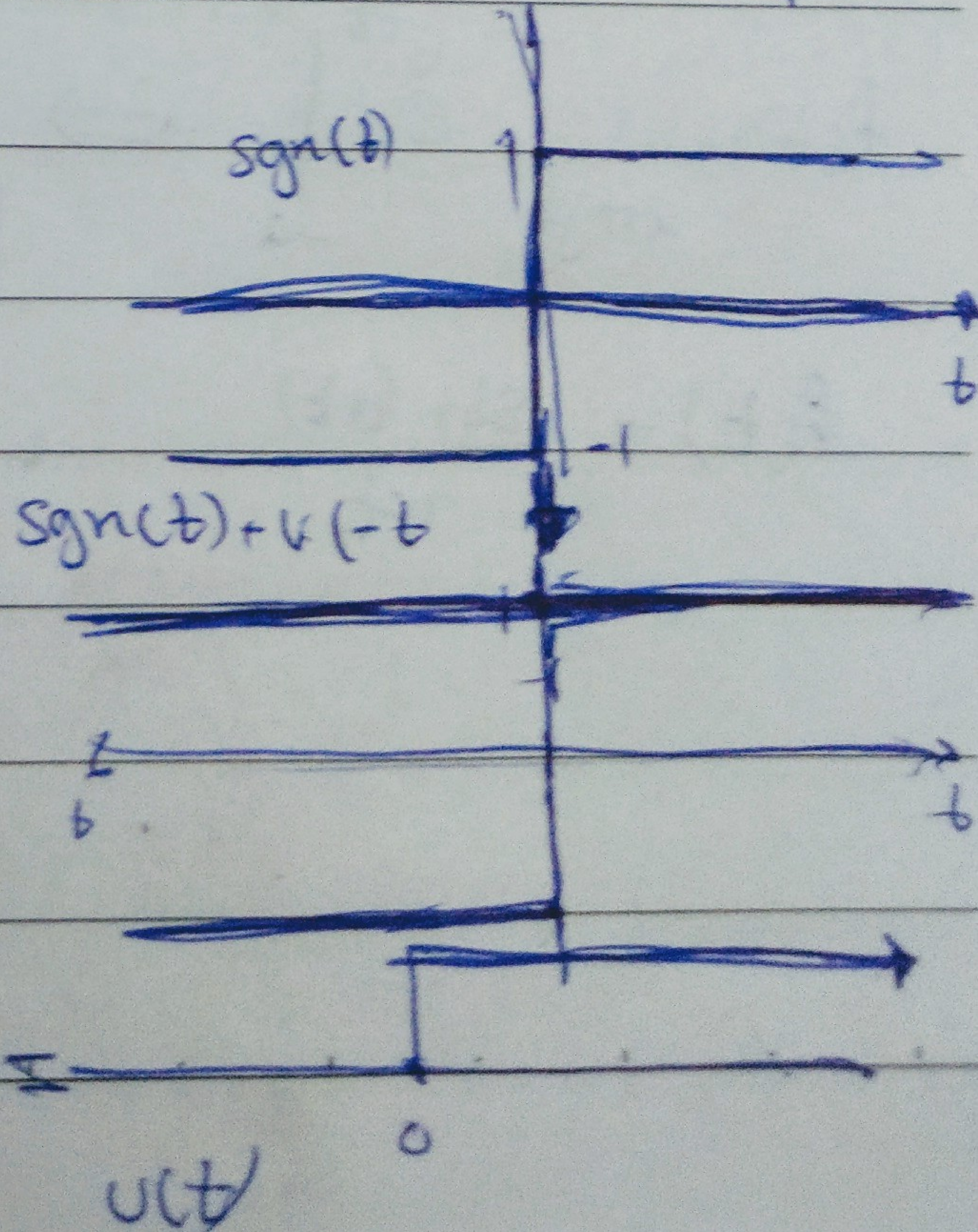
$3\,\mathrm{rect}\left(\frac{t}{8}\right) \xrightarrow{F.T} 24\,\mathrm{sinc}(4\omega).$



(c) $f(t) = \mathrm{sgn}(t) + u(-t)$

$f(t) = u(t)$

$F(\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$



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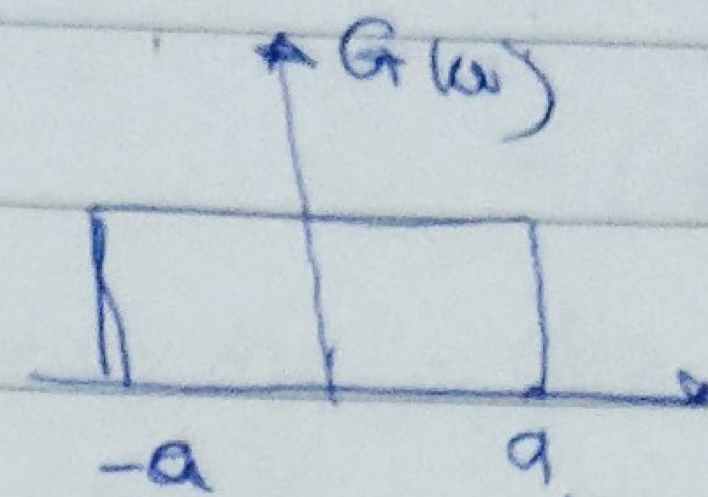
Q18 :-



ⓐ $G(\omega) = \text{rect}\left(\frac{\omega}{2a}\right)$

ⓑ We can find $g(t)$ using duality of direct integration

* By duality:

$$A\,\text{rect}\left(\frac{t}{2a}\right) \Longrightarrow 2aA\,\frac{\sin(\omega a)}{\omega a}$$

$$\cancel{2aA}\,\frac{\sin(at)}{\cancel{a}t} \xrightarrow{F.T} \cancel{A}2\pi\,\text{rect}\left(\frac{-\omega}{2a}\right)$$

$$\frac{\sin(at)}{t} \xrightarrow{F.T} \pi\,\text{rect}\left(\frac{\omega}{2a}\right)$$

$$\frac{\sin(at)}{\pi t} \xrightarrow{F.T} \text{rect}\left(\frac{\omega}{2a}\right)$$

So $f(t) = \frac{\sin(at)}{\pi t}$ ✓, or $\frac{a}{\pi}\,\text{sinc}(at)$

* By direct integration

$$f(t) = \frac{1}{2\pi}\int_{-a}^{a} 1 \cdot e^{j\omega t}\,d\omega$$

$$f(t) = \frac{1}{2\pi jt}\, e^{j\omega t}\Big|_{-a}^{a} \Longrightarrow \frac{1}{(\pi t)\,2j}\left[e^{ajt} - e^{-ajt}\right]$$

$$f(t) = \frac{\sin(at)}{\pi t}$$ ✓

Q 19:- $G(j\omega) = \frac{4}{j\omega + 6}$

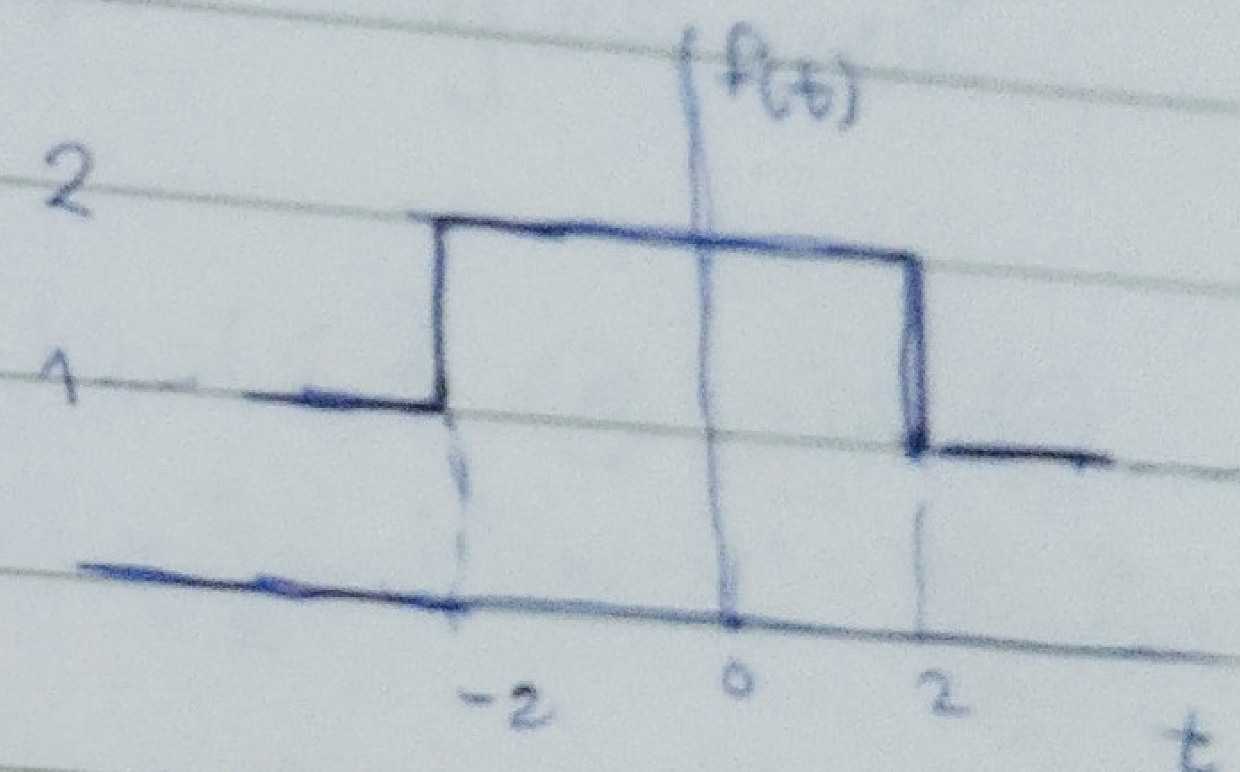
$x(t) = \frac{d^2 g(t)}{dt^2}$

F.T $\Downarrow$ F.T $\Downarrow$

$X(\omega) = (j^2 \omega^2) \cdot \frac{4}{j\omega + 6}$

$X(\omega) = \frac{-4\omega^2}{j\omega + 6}$

Q 20:-



$f(t) = 1 + \text{rect}(\frac{t}{4})$

$A = 1, a = 2$

$F(\omega) = \frac{4 \sin(2\omega)}{2\omega} + 2\pi\,\delta(\omega) = 4\,\text{Sin}(2\omega) + 2\pi\,\delta(\omega)$

Q 21:- $y(t) = f(t) \cdot \cos(100t)$? , $f(t) = \frac{2}{4 + jt}$

We can solve it using duality

$e^{-at} u(t) \xrightarrow{F.T} \frac{1}{a + j\omega}$

$\frac{1}{a + jt} \xrightarrow{F.T} 2\pi e^{-a(-\omega)} u(-\omega)$

$\frac{1}{a + jt} \xrightarrow{F.T} 2\pi e^{a\omega} u(-\omega)$, $a = 4$

$\frac{1}{4 + jt} \xrightarrow{F.T} 2\pi e^{4\omega} u(-\omega)$

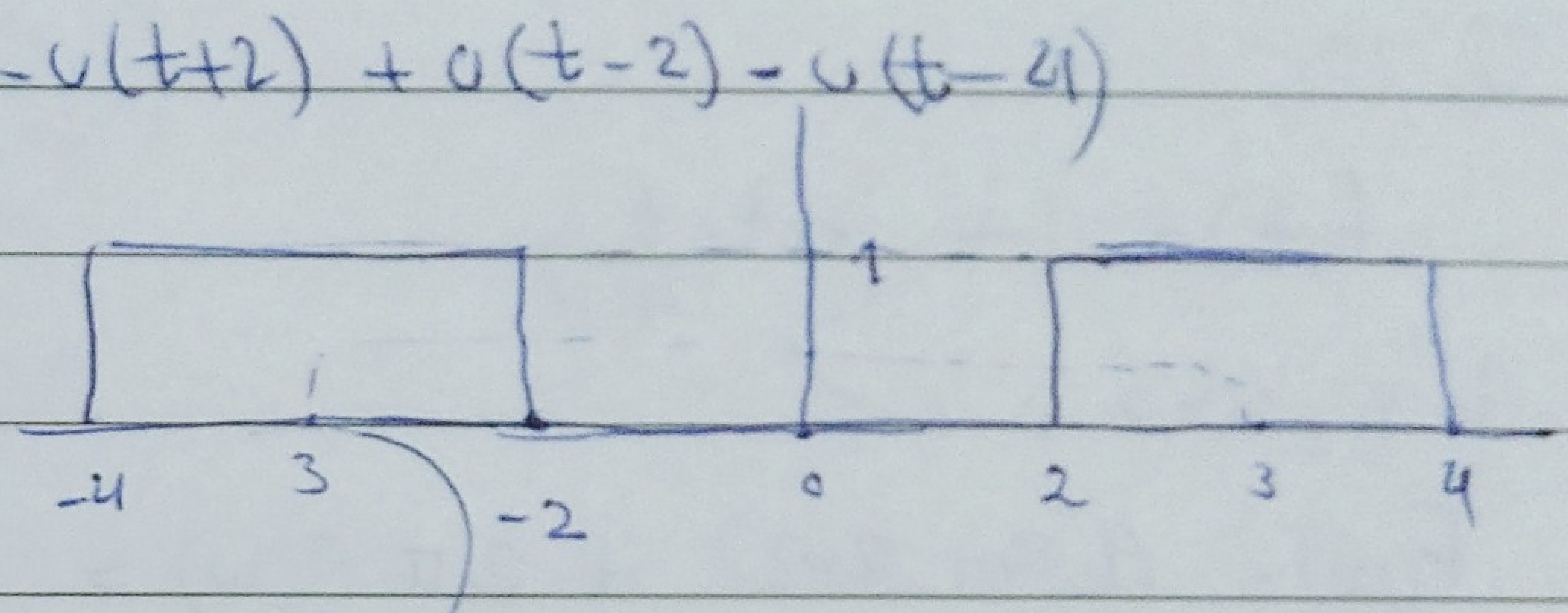
$$y(t) = f(t)\cos(100t)$$

$$Y(\omega) = \frac{1}{2}F(\omega-100) + \frac{1}{2}F(\omega+100)$$

$$Y(\omega) = \frac{1}{2}\left[4\pi e^{4(\omega-100)} u(-(\omega-100))\right] + \frac{1}{2}\left[4\pi e^{4(\omega+100)} u(-(\omega+100))\right]$$

$$Y(\omega) = 2\pi\left[e^{4(\omega-100)} u(-\omega+100) + e^{4(\omega+100)} u(-\omega-100)\right]$$

Q 22 - (a) $f(t) = u(t+4) - u(t+2) + u(t-2) - u(t-4)$



$$f(t) = \text{rect}\left(\frac{t+3}{2}\right) + \text{rect}\left(\frac{t-3}{2}\right)$$

$A = 1,\ a = 1,\ 2a = 2$

$$F(\omega) = \frac{2\sin(\omega)}{\omega}e^{3j\omega} + \frac{2\sin(\omega)}{\omega}e^{-3j\omega}.$$

Q22 $\quad X(j\omega) = u(\omega) \Rightarrow$ find $x(t)$

$$u(t) \xrightarrow{F.T} \frac{1}{j\omega} + \pi\delta(\omega)$$

$$u(-t) \xrightarrow{F.T} \frac{-1}{j\omega} + \pi\delta(-\omega)$$

$$u(-t) \xrightarrow{F.T} \frac{-1}{j\omega} + \pi\delta(\omega)$$

$$\frac{-1}{jt} + \pi\delta(t) \Rightarrow 2\pi u(\omega)$$

$$\frac{-1}{2\pi jt} + \frac{\delta(t)}{2} \longrightarrow u(\omega).$$

$$\frac{j}{2\pi t} + \frac{\delta(t)}{2} \Rightarrow U(\omega)$$

So the inverse fourier transform of of $U(\omega) = \frac{j}{2\pi t} + \frac{\delta(t)}{2}$

Q 23 :- $y(t) = A\left[1 + k\, m(t)\right] \cos(\omega_0 t)$

$$y(t) = A\cos(\omega_0 t) + A k\, m(t) \cos(\omega_0 t)$$

$$A \xrightarrow{F.T} 2\pi A\, \delta(\omega)$$

$$A k\, m(t) \xrightarrow{F.T} A k\, M(\omega)$$

$$Y(\omega) = \frac{\cancel{2}\pi A}{\cancel{2}} \delta(\omega - \omega_0) + \frac{\cancel{2}\pi A}{\cancel{2}} \delta(\omega + \omega_0) + \frac{A k}{2} M(\omega - \omega_0) + \frac{A k}{2} M(\omega + \omega_0)$$

$$Y(\omega) = \frac{A}{2}\left[2\pi\left(\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right) + k\left(M(\omega - \omega_0) + M(\omega + \omega_0)\right)\right]$$

① $r(t) = \frac{1}{1+t^2}$

$e^{-a|t|} \rightarrow \frac{2a}{a^2+\omega^2}$

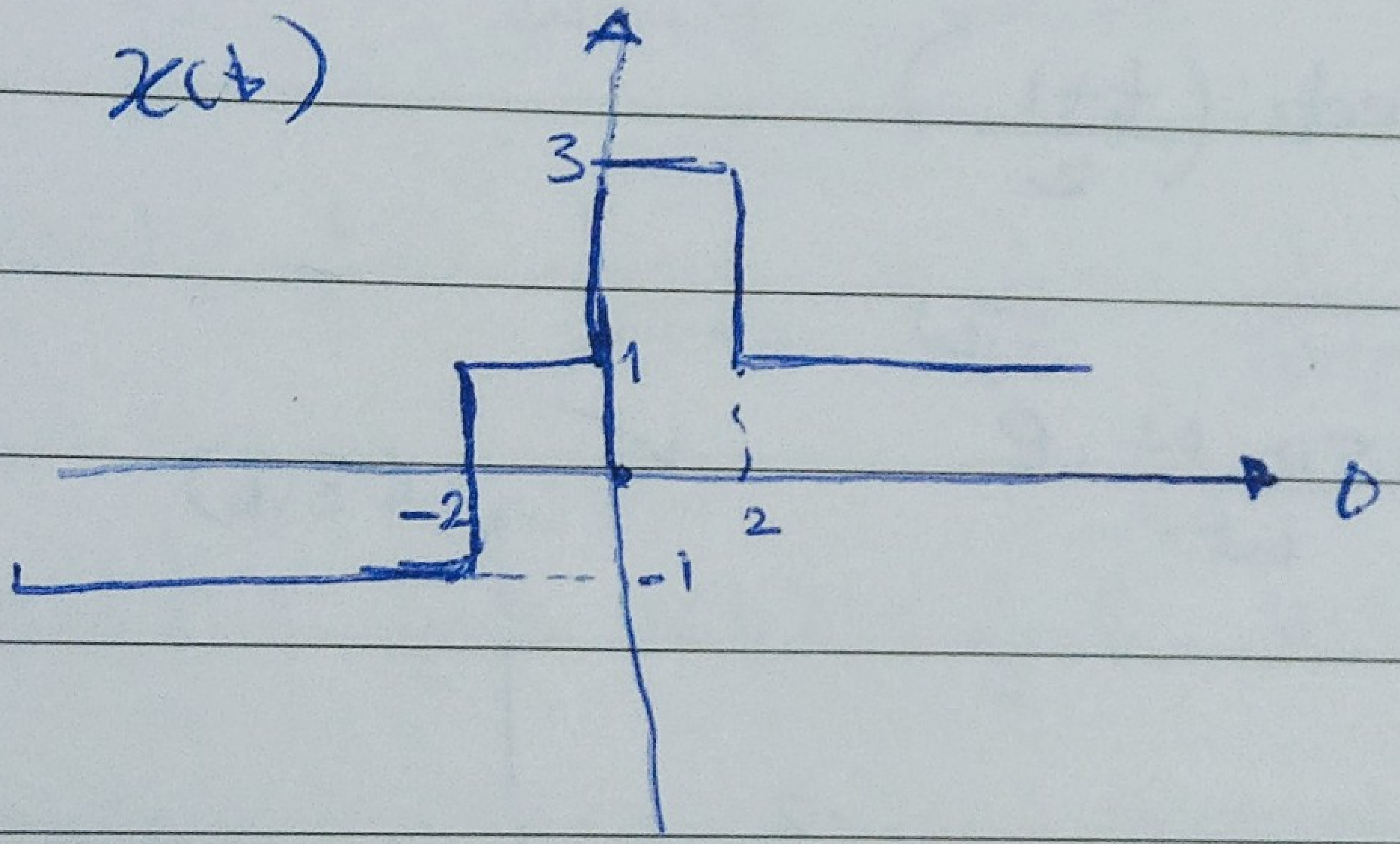
$\frac{2a}{a^2+t^2} \Rightarrow 2\pi e^{-a|-\omega|}$

$\frac{2a}{a^2+t^2} \Rightarrow 2\pi e^{-a|\omega|}$

For $a=1$

$\frac{2}{1+t^2} \Rightarrow 2\pi e^{-a|\omega|}$

$\frac{1}{1+t^2} \Rightarrow \pi e^{-a|\omega|}$



$x(t) = -u(-t+2) + u(t+2) + 2u(t) - 2u(t-2)$

$X(\omega) = \left(-\frac{1}{-j\omega} - \pi\delta(\omega)\right)e^{2j\omega} + \left(\frac{1}{j\omega} + \pi\delta(\omega)\right)e^{2j\omega}$

$+ \frac{2}{j\omega} + 2\pi\delta(\omega) - \frac{2}{j\omega}e^{-2j\omega} - 2\pi\delta(\omega)e^{-2j\omega}$

$= \frac{1}{j\omega}e^{2j\omega} + \frac{1}{j\omega}e^{2j\omega} + \frac{2}{j\omega} - \frac{2}{j\omega}e^{-2j\omega} - \cancel{\pi\delta(\omega)e^{2j\omega}} + \cancel{\pi\delta(\omega)e^{2j\omega}}$

$+ 2\pi\delta(\omega) - \cancel{2\pi\delta(\omega)e^{-2j\omega}}$ ①

$= \frac{2}{j\omega}e^{2j\omega} + \frac{2}{j\omega} - \frac{2}{j\omega}e^{-2j\omega}$ $\cancel{\frac{2}{j\omega}e^{-j\omega}}$

or $x(t) = \mathrm{Sgn}(t+2) + 2\,\mathrm{rect}\left(\frac{t-1}{2}\right)$

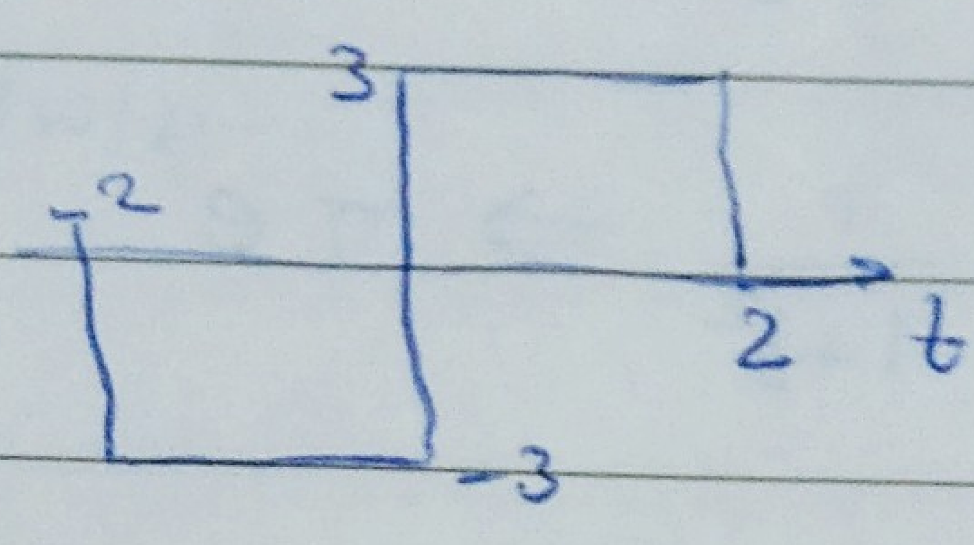
$X(j\omega) = \frac{2}{j\omega} e^{2j\omega} + 4\frac{\sin(\omega)}{\omega} e^{-j\omega}$

$X(\omega) = \frac{2}{j\omega} e^{2j\omega} + \left(\frac{2e^{j\omega}}{j\omega} - \frac{2e^{-j\omega}}{j\omega}\right) e^{-j\omega}$

$X(\omega) = \frac{2}{j\omega} e^{2j\omega} + \frac{2}{j\omega} - \frac{2e^{-2j\omega}}{j\omega}$ ✓

~~or $\frac{d}{} x(t) = \delta(t)$~~

(b)

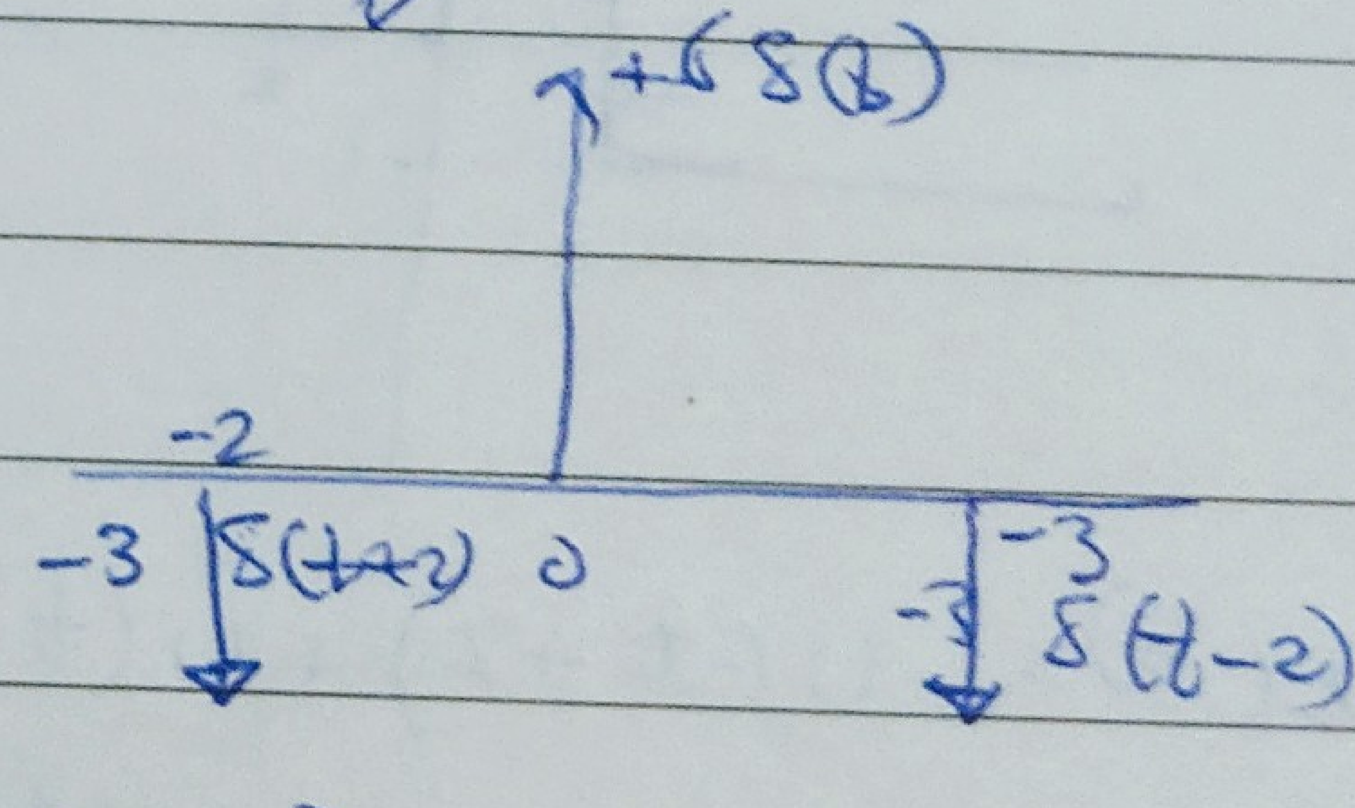


$y(t) = 3\,\mathrm{ret}\left(\frac{t-1}{2}\right) - 3\,\mathrm{rect}\left(\frac{t+1}{2}\right)$

$= 6\frac{\sin(\omega)}{\omega} e^{-j\omega} - 6\frac{\sin\omega}{\omega} e^{j\omega}$ ✓

$=$

~~$= \frac{3}{j\omega}\left(e^{j\omega} - e^{-j\omega}\right)$~~



or $\frac{d(x)}{dt} = -3\delta(t+2) + 6\delta(t) - 3\delta(t-2)$

$j\omega X(\omega) = -3e^{2j\omega} + 6 - 3e^{-2j\omega}$

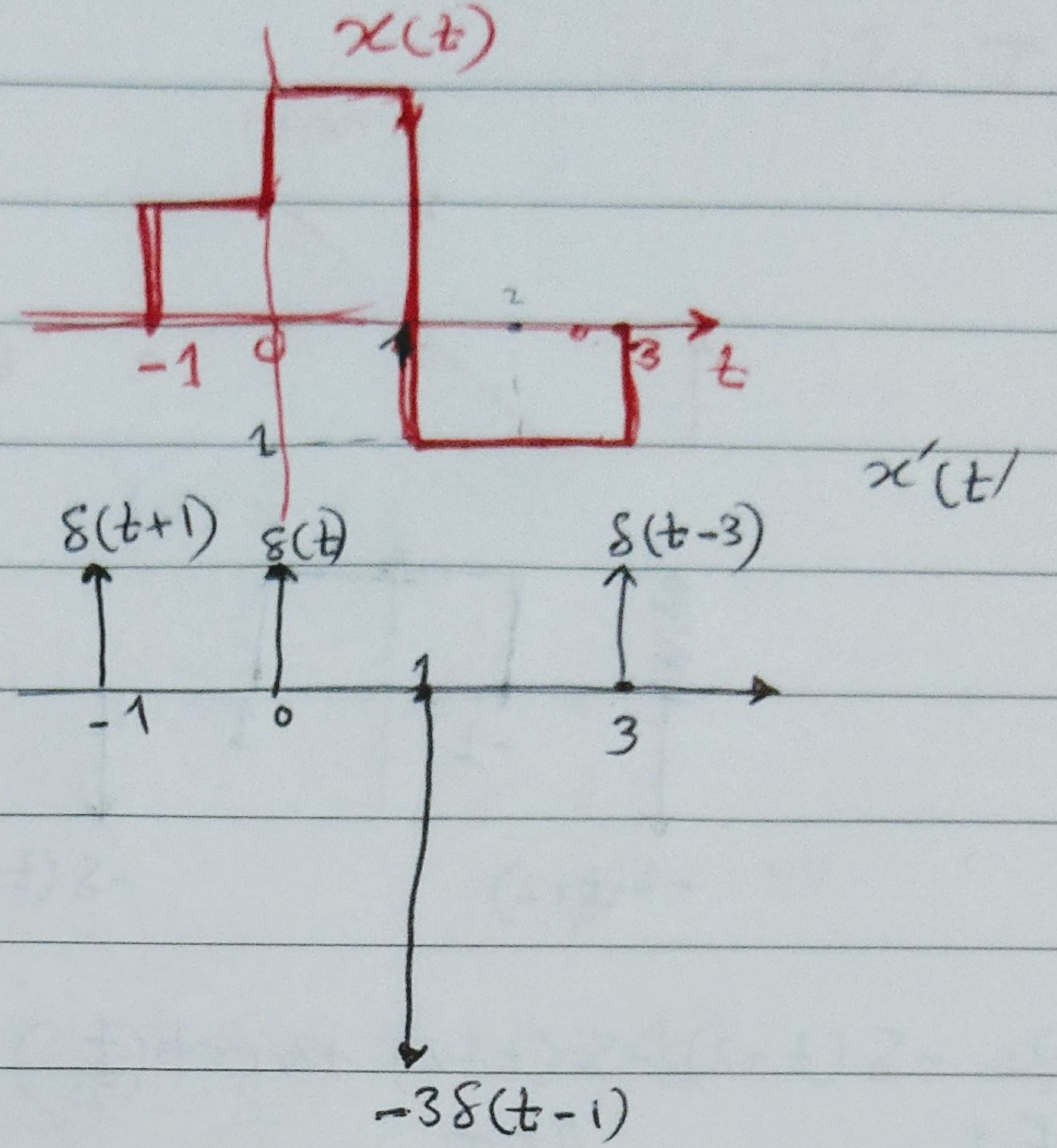
$\Rightarrow X(\omega) = \frac{-3e^{2j\omega} - 3e^{-2j\omega} + 6}{j\omega} = \frac{6(1-\cos 2\omega)}{j\omega} = \frac{12\sin^2\omega}{j\omega}$

$= 6\frac{\sin\omega}{\omega}\left(e^{-j\omega} - e^{j\omega}\right) = \frac{-6\sin\omega}{\omega} \cdot 2j\sin\omega$

$= \frac{12\sin^2\omega}{j\omega}$ ~~$= \frac{12(1-\cos 2\omega)}{2j\omega} = 6t$~~

Find F.T for :-

(a)



$$x'(t) = \delta(t+1) + \delta(t) - 3\delta(t-1) + \cancel{2}\delta(t-3)$$

F.T $\Downarrow$ F.T $\Downarrow$

$$j\omega X(\omega) = e^{j\omega} + 1 - 3e^{-j\omega} + e^{-3j\omega}$$

$$X(\omega) = \frac{\left[1 + e^{j\omega} - 3e^{-j\omega} + e^{-3j\omega}\right]}{j\omega}$$

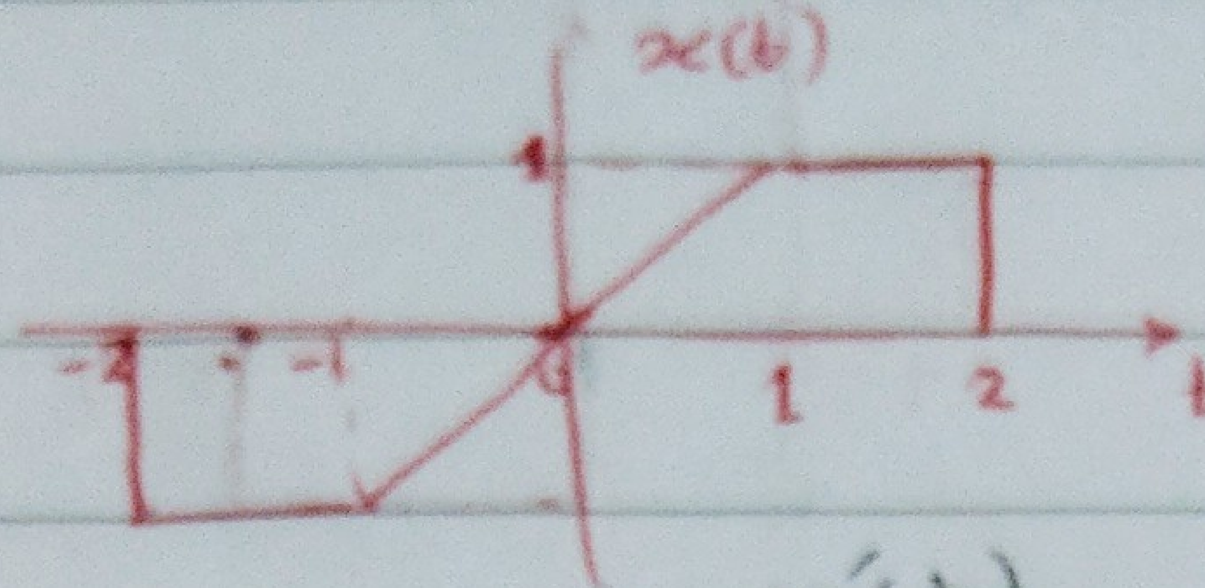
or = $x(t) = \text{rect}\left(\frac{t}{2}\right) + \text{rect}\left(t - \frac{1}{2}\right) - \text{rect}\left(\frac{t-2}{2}\right)$

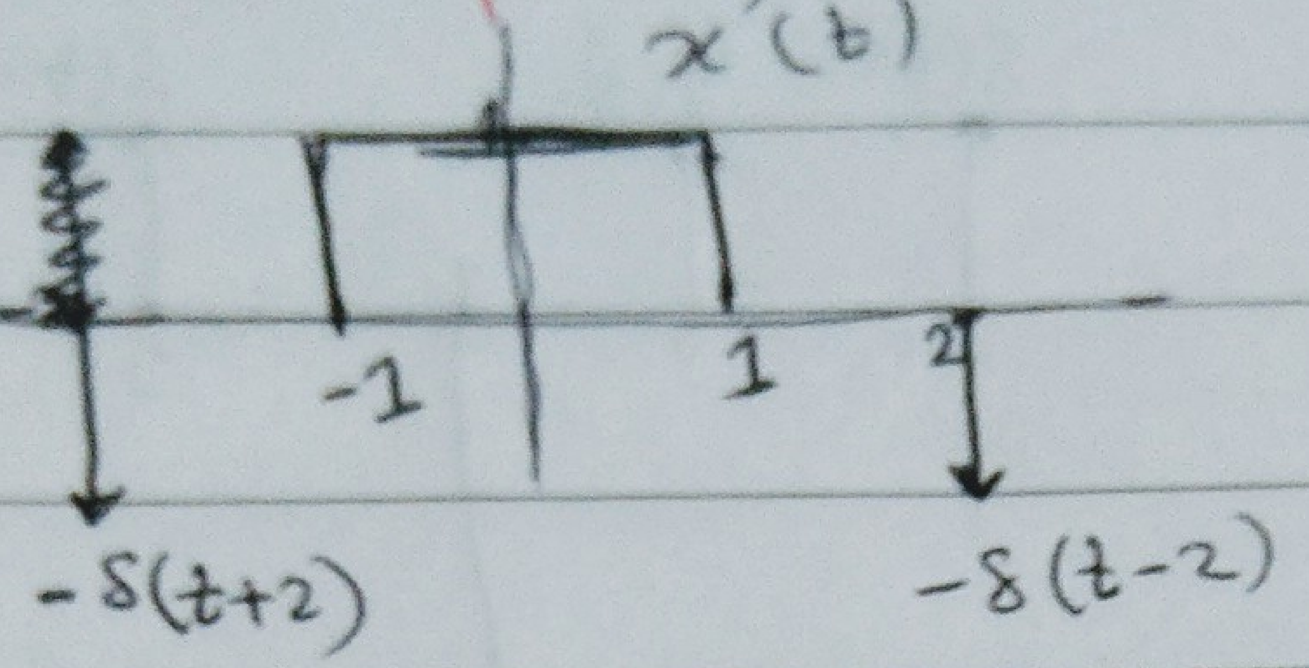
$$X(\omega) = 2\frac{\sin(\omega)}{\omega} + \frac{\sin\left(\frac{\omega}{2}\right)}{\frac{\omega}{2}} e^{-\frac{j\omega}{2}} - 2\frac{\sin(\omega)}{\omega} e^{-2j\omega}$$

$$X(\omega) = \frac{2}{\omega}\left[\frac{e^{j\omega} - e^{-j\omega}}{2j}\right] + \frac{2}{\omega}\left[\frac{e^{\frac{j\omega}{2}} - e^{-\frac{j\omega}{2}}}{2j}\right] e^{-\frac{j\omega}{2}} - \frac{2}{\omega}\left[\frac{e^{j\omega} - e^{-j\omega}}{2j}\right] e^{-2j\omega}$$

$$X(\omega) = \frac{e^{j\omega} - e^{-j\omega} + 1 - e^{-j\omega} - e^{-j\omega} + e^{-3j\omega}}{j\omega} = \frac{1 + e^{j\omega} - 3e^{-j\omega} + e^{-3j\omega}}{j\omega}$$

Find F.T of:-





$$x'(t) = -\delta(t+2) - \delta(t-2) + 2\,\mathrm{rect}\left(\frac{t}{2}\right)$$

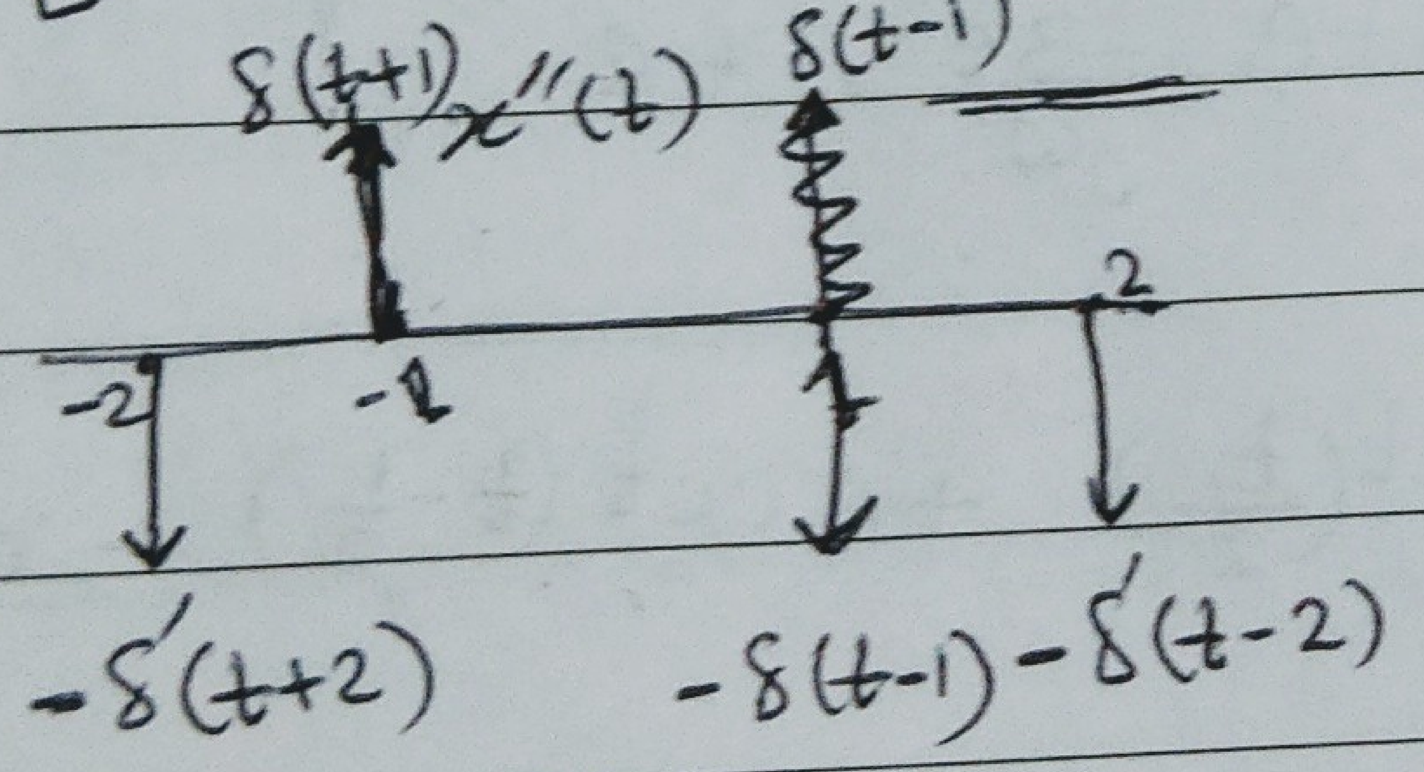
$\Downarrow$ F.T $\qquad\qquad \Downarrow$ F.T

$$(j\omega)X(\omega) = -e^{2j\omega} - e^{-2j\omega} + \frac{2\sin\omega}{\omega}$$

$$= -2\left(\frac{e^{2j\omega} + e^{-2j\omega}}{2}\right) + \frac{2\sin\omega}{\omega}$$

$$\cancel{jX(\omega)} \quad X(\omega) = \frac{-2}{j\omega}\left[\cos 2\omega - \frac{\sin\omega}{\omega}\right]$$

or



$$x''(t) = -\delta'(t+2) + \delta(t+1) - \delta(t-1) - \delta'(t-2)$$

$\Downarrow$

$$(j\omega)^2 X(\omega) = -j\omega e^{2j\omega} + e^{j\omega} - e^{-j\omega} - j\omega e^{-2j\omega}$$

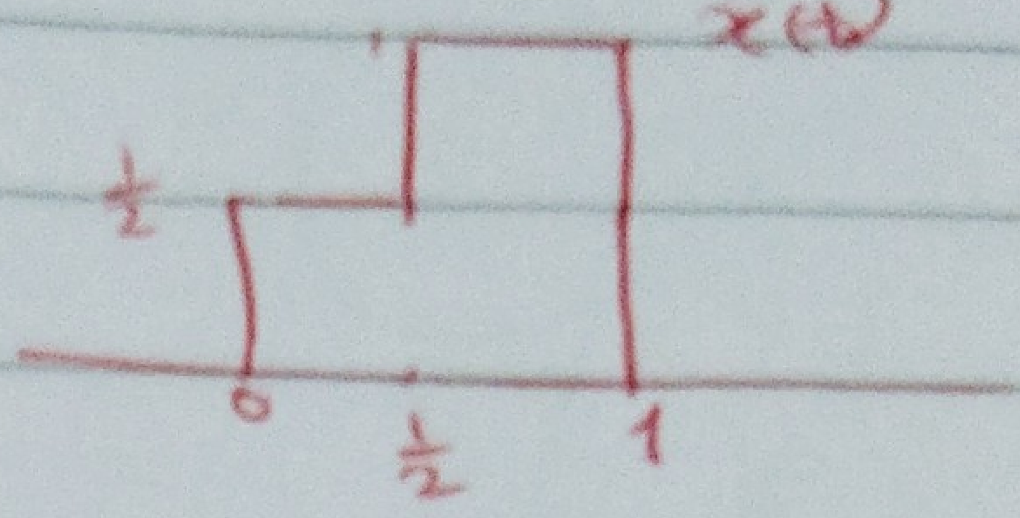
$$X(\omega) = \frac{1}{j^2\omega^2}\left[-j\omega e^{2j\omega} - j\omega e^{-2j\omega} + e^{j\omega} - e^{-j\omega}\right]$$

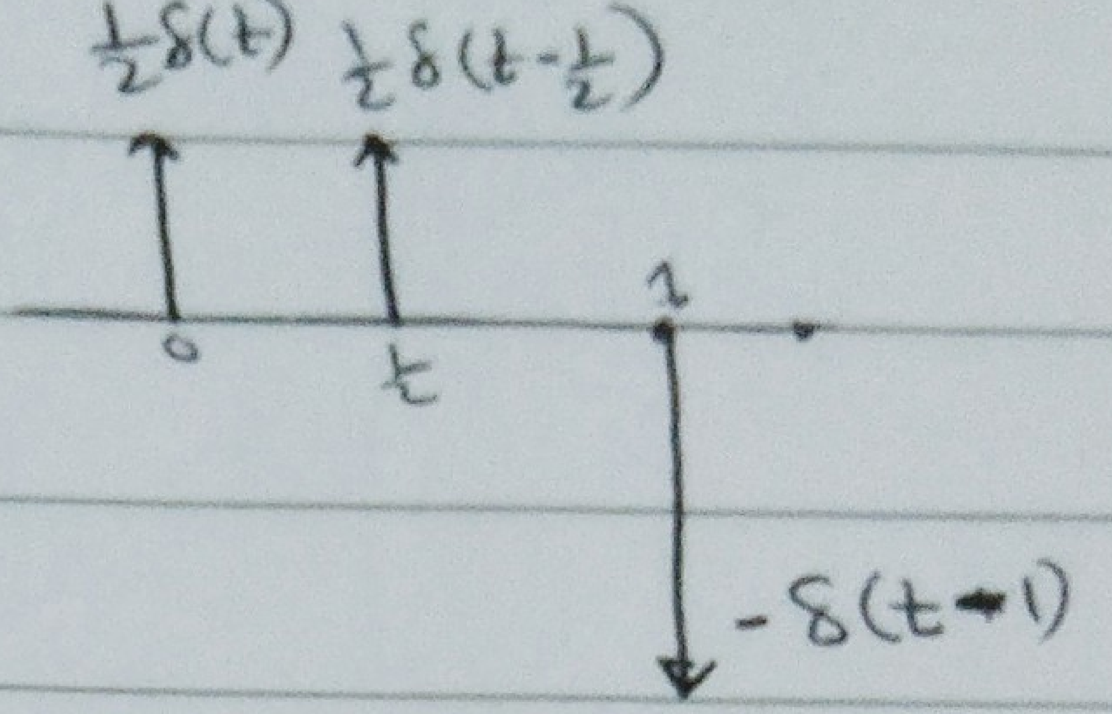
$$= \frac{-2}{j\omega}\left[\frac{j\omega e^{2j\omega} + j\omega e^{-2j\omega}}{2j\omega} - \frac{(e^{j\omega} - e^{-j\omega})}{2j\omega}\right]$$

$$X(\omega) = \frac{-2}{j\omega}\left[\cos 2\omega - \frac{\sin\omega}{\omega}\right]$$

✓

Find F.T of



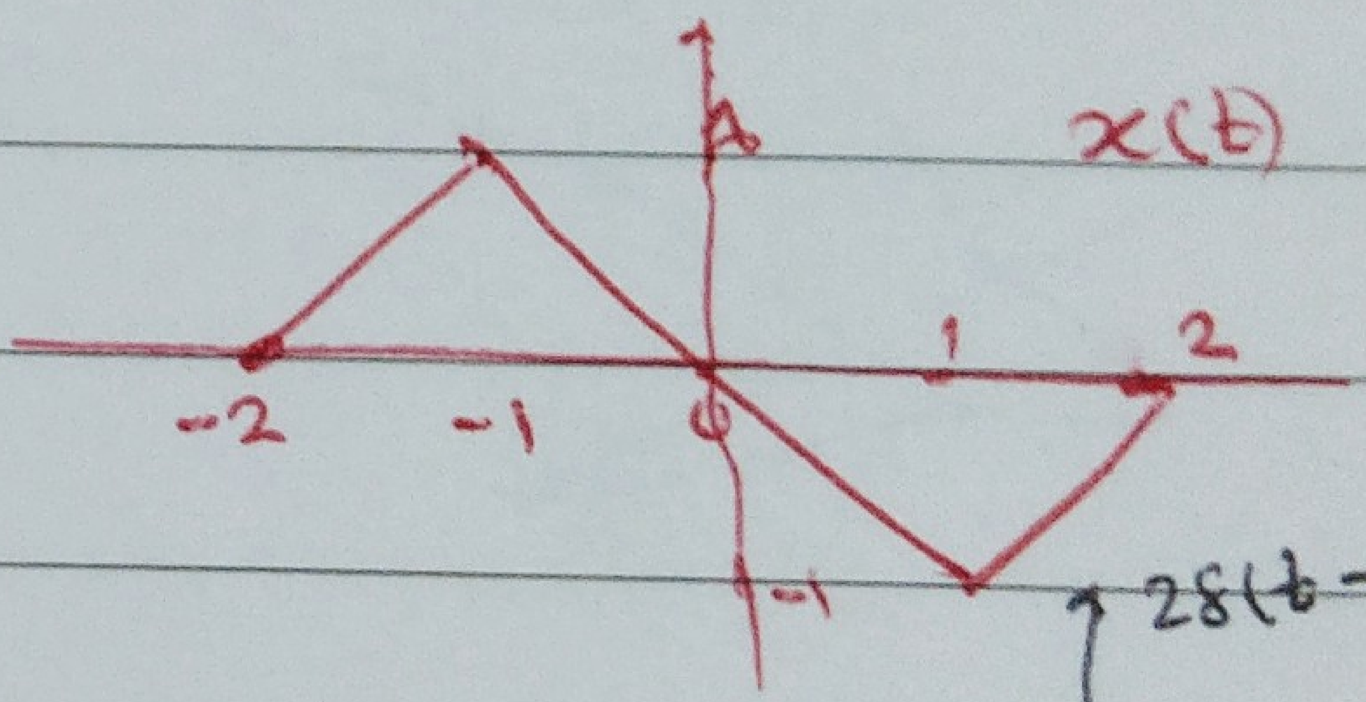


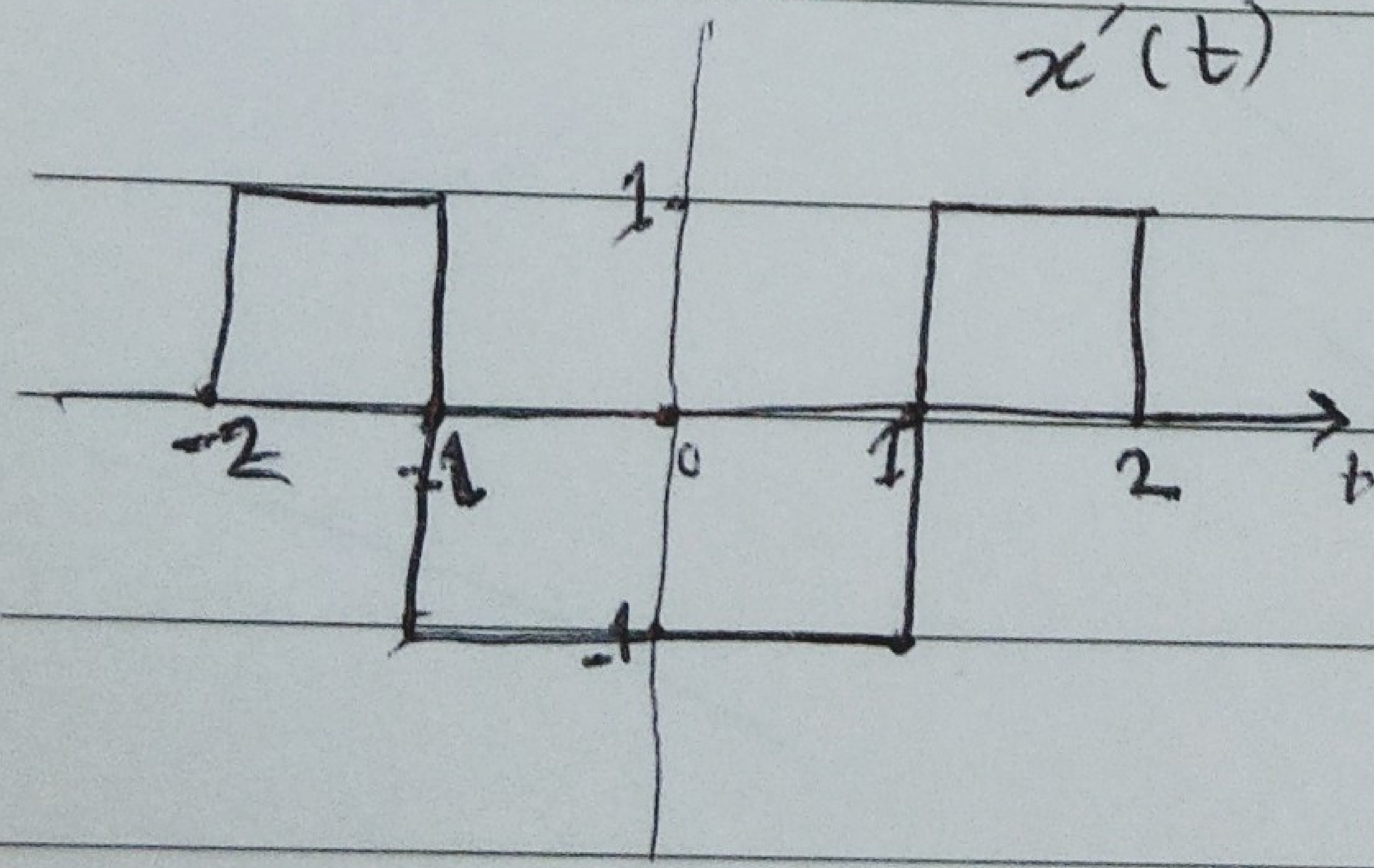
$$x'(t) = \frac{1}{2}\delta(t) + \frac{1}{2}\delta\left(t-\frac{1}{2}\right) - \delta(t-1)$$

$$(j\omega)X(\omega) = \frac{1}{2} + \frac{1}{2}e^{-\frac{j\omega}{2}} - e^{-j\omega}$$

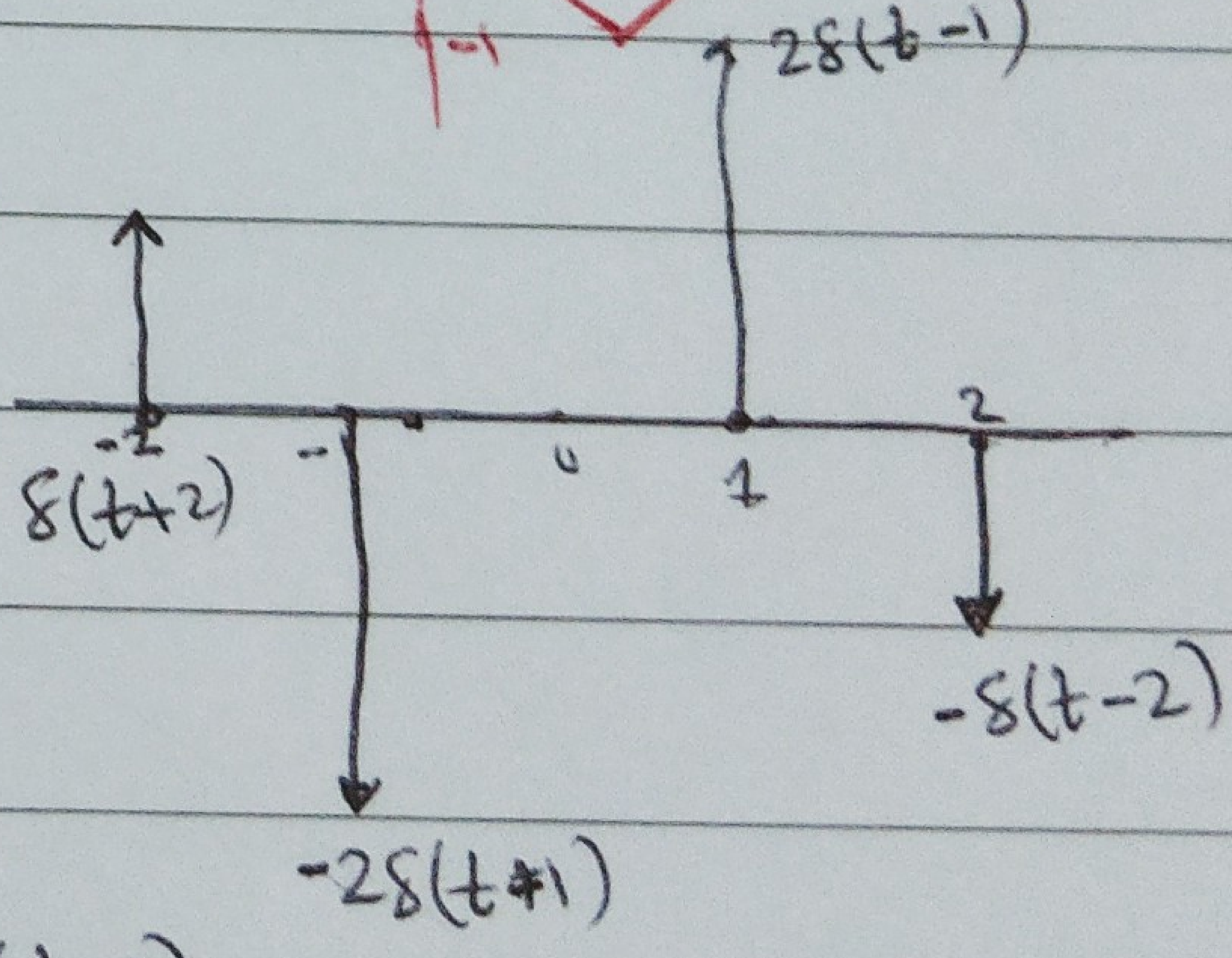
$$X(\omega) = \left[\frac{1}{2} + \frac{1}{2}e^{-j\frac{\omega}{2}} - e^{-j\omega}\right]\cdot\frac{1}{j\omega}$$

Find F.T of:-





$\Rightarrow$



$$x''(t) = \delta(t+2) - \delta(t-2) - 2\delta(t+1) + 2\delta(t-1)$$

$$(j\omega)^2 X(\omega) = e^{2j\omega} - e^{-2j\omega} - 2e^{j\omega} + 2e^{-j\omega}$$

$$X(\omega) = \frac{e^{2j\omega} - e^{-2j\omega}}{j^2\omega^2} - 2\left(\frac{e^{j\omega} - e^{-j\omega}}{j^2\omega^2}\right) = \frac{2\sin(2\omega)}{j\omega^2} - \frac{4\sin\omega}{j\omega^2}$$

$$= \frac{4}{j\omega}\left[\operatorname{sinc}(2\omega) - \operatorname{sinc}(\omega)\right]$$

If for a given system $x(t) = e^{-t}u(t)$ and $h(t) = e^{-2t}u(t)$ find $y(t)$.

$$X(s) = \frac{1}{s+1} \quad , \quad H(s) = \frac{1}{s+2}$$

$$Y(s) = X(s) \cdot H(s) = \frac{1}{s+1} \cdot \frac{1}{s+2}$$

$$Y(s) = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = \left.\frac{1}{s+2}\right|_{s=-1} = \frac{1}{1} = 1 \quad , \quad B = \left.\frac{1}{s+1}\right|_{s=-2} = \frac{1}{-1} = -1$$

$$Y(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

$$y(t) = (e^{-t} - e^{-2t})u(t)$$

Solve :- $\frac{dy(t)}{dt} + 4y(t) = x(t)$, $y(0^-) = 0$, $x(t) = e^{-2t}u$

$$sY(s) - y(0^-) + 4Y(s) = \frac{1}{s+2}$$

$$(s+4)Y(s) = \frac{1}{s+2} \Rightarrow Y(s) = \frac{1}{(s+4)(s+2)} = \frac{A}{s+4} + \frac{B}{s+2}$$

$$A = \left.\frac{1}{s+2}\right|_{s=-4} = \frac{1}{-2} = -\frac{1}{2} \quad , \quad B = \left.\frac{1}{s+4}\right|_{s=-2} = \frac{1}{2}$$

$$Y(s) = \left(\frac{1}{2}e^{-2t} - \frac{1}{2}e^{-4t}\right)u(t)$$

Find I.L.T of:- $X(s') = \frac{s'+5}{s'^2+6s'+8}$

$$X(s') = \frac{s'+5}{(s'+4)(s'+2)} = \frac{A}{s'+4} + \frac{B}{s'+2}$$

$$A = \left.\frac{s'+5}{s'+2}\right|_{s'=-4} = \frac{-4+5}{-4+2} = -\frac{1}{2}$$

$$B = \left.\frac{s'+5}{s'+4}\right|_{s'=-2} = \frac{-2+5}{-2+4} = \frac{3}{2}$$

$$X(s') = \frac{3}{2} \cdot \frac{1}{s'+2} - \frac{1}{2} \cdot \frac{1}{s'+4}$$

$$x(t) = \left(\frac{3}{2}e^{-2t} - \frac{1}{2}e^{-4t}\right)u(t).$$

Find I.L.T of $\dfrac{s^2 - s + 1}{s^2 (s-1)}$

$$X(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1}$$

$$C = \left.\frac{s^2 - s + 1}{s^2}\right|_{s=1} = \frac{1-1+1}{1} = 1$$

$$B = \left.\frac{s^2 - s + 1}{(s-1)}\right|_{s=0} = \frac{1}{-1} = -1$$

$$A = \left.\frac{d}{ds}\left(\frac{s^2 - s + 1}{s-1}\right)\right|_{s=0} = \left.\frac{(2s-1)(s-1) - s^2 + s - 1}{(s-1)^2}\right|_{s=0} = \frac{(-1)(-1) - 1}{(-1)^2}$$

$$A = \frac{1-1}{1} = 0$$

$$X(s) = \frac{-1}{s^2} + \frac{1}{s-1}$$

$$x(t) = -t\,u(t) + e^{-t}\,u(t) = (e^{\cancel{t}} - t)\,u(t)$$

Find I.L.T of $\dfrac{s^2 - s + 1}{(s+1)^2}$ البسط والمقام من نفس الدرجة

$$X(s) = \frac{s^2 - s + 1 + 2s - 2s}{s^2 + 2s + 1} = \frac{s^2 + 2s + 1 - 3s}{s^2 + 2s + 1}$$

$$X(s) = \frac{s^2 + 2s + 1}{s^2 + 2s + 1} - \frac{3s}{s^2 + 2s + 1} = 1 - \frac{3s}{(s+1)^2}$$

$$X(s) = 1 - \left(\frac{A}{(s+1)} + \frac{B}{(s+1)^2}\right)$$

$\cancel{A = 3s}$ $A(s+1) + B = 3s$

$As + A + B = 3s$

$As = 3s \Rightarrow A = 3$

$A + B = 0 \Rightarrow B = -3$

$$X(s) = 1 - \frac{3}{(s+1)} + \frac{3}{(s+1)^2}$$

$$x(t) = \delta(t) - 3e^{-t}u(t) + 3te^{-t}u(t)$$

Find I.L.T of :- $J(s) = \frac{12}{(s+2)^2(s+4)}$

$$J(s) = \frac{A}{s+4} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$A = \left.\frac{12}{(s+2)^2}\right|_{s=-4} = \frac{12}{(-2)^2} = 3$$

$$C = \left.\frac{12}{s+4}\right|_{s=-2} = \frac{12}{-2+4} = \frac{12}{2} = 6$$

$$B = \left.\frac{d}{ds}\left(\frac{12}{s+4}\right)\right|_{s=-2} = \left.\frac{-12}{(s+4)^2}\right|_{s=-2} = \frac{-12}{(-2)^2} = -3$$

$$J(s) = \frac{3}{s+4} - \frac{3}{s+2} + \frac{6}{(s+2)^2}$$

$$j(t) = \left(3e^{-4t} - 3e^{-2t} + 6te^{-2t}\right)u(t)$$

* A continuous time system is given by:

$$\frac{d^2 y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 6y(t) = \frac{dx(t)}{dt} - 4x(t)$$

$y'(0^-) = 0 \quad y(0) = 0 , \ x(0) = 0$

* Find H(s) and impulse response h(t)?

$$s^2 Y(s) + 5 s Y(s) + 6 Y(s) = s X(s) - 4 X(s)$$

$$Y(s)\left[s^2 + 5s + 6\right] = X(s)\left[s - 4\right]$$

$$\frac{Y(s)}{X(s)} = \frac{s-4}{s^2+5s+6}$$

$$H(s) = \frac{s-4}{s^2+5s+6} = \frac{s-4}{(s+2)(s+3)}$$

$$H(s) = \frac{A}{s+2} + \frac{B}{s+3}$$

$$A = \left.\frac{s-4}{s+3}\right|_{s=-2} = \frac{-2-4}{-2+3} = \frac{-6}{1} = -6$$

$$B = \left.\frac{s-4}{s+2}\right|_{s=-3} = \frac{-3-4}{-3+2} = \frac{-7}{-1} = 7$$

$$H(s) = \frac{-6}{s+2} + \frac{7}{s+3}$$

$$h(t) = \left(7e^{-3t} - 6e^{-2t}\right)u(t)$$